Full Analysis of All Composite Patch Repairing Design Parameters

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Abstract:*Repairing a crack in a structure consists in reducing crack's tips stresses by transferring loads trough a bridge made of the composite patch and the adhesive. This operation is impacted by four factors: shear modulus of the adhesive, the composite patch's Young module and the thicknesses of these two materials. The design of experiments method allowed us to determine, the weight of each of the four factors and their interactions as well their best combination to obtain an efficient and lasting repair. The constraints relative to the stiffness ratio and the shear strain were taken into consideration in order to determine the best configuration that allowed the minimization of* K_∞.

Keywords: *crack, adhesive, composite patch, factors, repair.*

1. INTRODUCTION

There are two alternative approaches to fracture analysis: the energy criterion and the stress intensity approach. The energy approach states that crack extension occurs when the energy available for crack growth is sufficient to overcome the resistance of the material [1] [2].

Griffith [3] was the first to propose the energy criterion for fracture, but Irwin [4] is primarily responsible for developing the present version of this approach: the energy release rate, G, which is defined as the rate of change in potential energy within the crack area for a linear elastic material.

For a crack of length *2a,* in an infinite plate subject to a remote tensile stress σ, this energy release rate is given by

$$G=\frac{\pi\sigma^{2a}}{E} \tag{1}$$

E being the Young's modulus. As it can be seen this energy grows proportionally to the length of the crack .It constitutes the driving force for the fracture until it reaches a critical value G_c at which the structure collapses and which corresponds to a critical crack length a_c.

The stress intensity approach states that near the tip of a crack in an elastic material, when considering an in-plane tresses case, each stress component is proportional to a single constant K. If this constant is known, the entire stress distribution at the crack tip can be computed with the equations below.

$$K=Y\sigma\sqrt{\pi a} \tag{2}$$

Y is a geometrical constant.

This constant, which is called the stress-intensity factor, completely characterizes the crack-tip conditions in a linear elastic material.

A large litterature has been devoted to this subject. We can cite A. Hassani et al. who worked on the relation of crack growth with its critical size using damage tolerance concept [5]. A. Kotoussov et al. tackled the problem of local plastic collapse of a plate [6]. D. Chang et al. developed a general theoretical approach to investigate the fatigue behavior of two interacting cracks [7]. Haddad et al. determined the hot cracking susceptibility [8]. Lately Jun Ding et al. performed a molecular dynamics simulation of crack propagation in a single crystal aluminum plate with central cracks [9].

As stated earlier cracks grow until they reach a critical value at which the structure collapses. Therefore their repair at the appropriate time is mandatory. Crack repairing of structures consists in reducing crack's tips stresses by transferring loads trough a bridge made up of the composite patch and the adhesive. (Fig.1). Initial crack repairs were accomplished by attaching reinforcing plates over the damaged areas either by welding or by mechanical fasteners that used drilled holes. These methods worked well but each one of them had a big disadvantage. The first one induced non desirable residual stresses while the second created local stresses raisers.

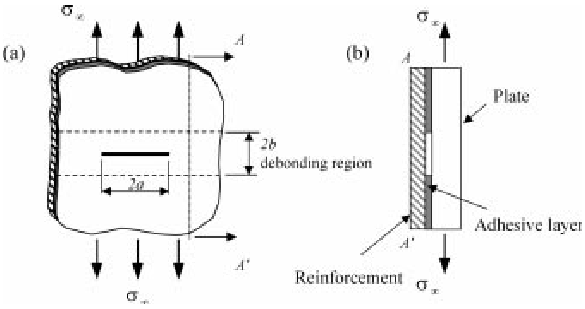

Fig 1: Patch configurations

In addition to eliminating these problems, composite patches are lightweight, relatively easy to implement and can adapt to almost any geometry. The concept of using composite patch repairs started in the early 1970's and was successfully applied in aircraft structures. The pioneers of this technique were Baker [10] [12], Jones [13] and Barthlomeusz [14]. Once cracks' structures repairing with composite patches became widely used, designers and researchers shifted their interest towards the optimization of this technique. Initial works were devoted to geometrical forms and dimensioning. Later on, more sophisticated methods that utilised algorithms were introduced. Among them we can cite Chue et al. [15] who studied the effects of ply orientation, Brighenti et al. [16] applied the genetic algorithm to the optimization of this technique ; Tsamasphyros et al. [17] used optic fibers which allowed simulations of different sequencing of piling in order to obtain their optimization. Ait Yala et al. [18] used the design of experiments method to obtain the most efficient configuration.

All approaches cited above are satisfactory and give good results. However they only take into consideration selected parameters that affect the composite patch repairs. This makes them somehow partial as several factors affect the effectiveness of composite patch repairing and its performance. The purpose of this work is to further these researches by proposing a full analysis of all the design parameters of this technique. This will not only allow us to select the most fitted composite patch and adhesive , but also to consider all the constraints in order to perform the most efficient, lasting and cost effective repair.

2. Method

According to the linear elastic fracture mechanics, for a plate with infinite size and a central crack (opening mode I) the stress intensity factor (SIF) is given by the following relation:

$$K_I = \sigma\sqrt{\pi a} \quad (3)$$

where σ is the applied stress and a half the length of the crack. One can see that K_I becomes larger as the length of the crack grows. When it reaches a certain value, the structure collapses. However after patch repairing, K_I tends towards a constant asymptote value K_∞ as the crack length increases. Indeed a well designed and bonded patch causes the stress intensity at the crack tip to reach a limiting factor, K_∞, no matter how long the crack length becomes. When fully bonded on one side, Rose [20] obtained an analytical approximate solution of this upper bound. This was based on the Rose model which is a continuum analysis based on the theory of elasticity. It covers the stress intensity solution, Kr, for the repaired crack, the adhesive shear strain in the bond line and the load attraction into the stiffened area.

The expression of this asymptote is given by the relation:

$$K_\infty = \sigma_0\sqrt{\pi\Lambda} \quad (4)$$

The expression of Kr for repaired structures is given by :

$$Kr = \sigma_0\sqrt{\frac{\pi a\Lambda}{a+\Lambda}} \quad (5)$$

And the expression of the maximum shear strain in the adhesive is:

$$\gamma_{max} = \sigma_0 t_p \frac{\beta}{Ga} \quad (6)$$

Where

$$\sigma_0 = \frac{tsEs}{tsEs + tpEp}\,\sigma$$

$$\Lambda = \; sqrt\left[\frac{tsEs}{\beta\left(1+\frac{tsEs}{tpEp}\right)}\right]$$

.

$$\text{With } \beta^2 = \left[\left(\frac{Ga}{ta}\right)\left(\frac{1}{Ests}+\frac{1}{Eptp}\right)\right]$$

E and G are respectively the modulus of elasticity and the shear modulus; t being the thickness of the material. The subscript's' refers to the structure (plate); 'p' to the patch (reinforcement) and 'a' to the adhesive.

Optimum design consists in determining design parameters (variables) that lead to the best performances. Therefore we must find the best configuration that allows the minimization of K_∞. It is quite evident that by lowering this asymptote we will lower the stress intensity factors and thus obtain an optimum design. We choose to consider K_∞ instead of Kr because the length of the crack a does not appear in its expression (Eq.2) which will confer more generality to our results.

A close look to this function shows that all the ingredients necessary to perform this task are contained within this expression. In deed Es, ts and Gs are the geometrical and physical properties of the structure to be repaired. Thus they are the given of the problem which consists in selecting the appropriate composite patch and adhesive materials properties (Ep, Gp, Ea and Ga) as well as their dimensions (tp and ta) in order to achieve the best design by obtaining the lowest value of K_∞. In other words these are our design variables, and the most effective method which will allow us to evaluate and analyze them is the design of experiments method. This statistical method is well presented by Benoist et al [20]; Fischer R. A [21] and Taguchi,et al.[22]. Indeed it allowed us to obtain the weight of all the factors and their interactions as well as their most effective combination to achieve the lowest K_∞.

However selecting the best parameters values for K_∞ alone could not be satisfactory as there are two important aspects that a designer should take into consideration:

- The load transfer criteria which define the stiffness ratio that ensures a good transfer of load. It is defined by SR= $E_p t_p / E_s t_s$ and should be kept close to unity.

- The maximum shear strain in the adhesive, γ_{max} given in eq. (6) must be kept at its lowest possible value.

As it can be can seen both of these entities depend upon the design parameters and the final elected values should not only permit obtaining the minimum K_∞ but also satisfy these two last constraints.

3. RESULTS AND DISCUSSION

The expressions of K_∞, γ_{max} and the stiffness ratio show that there are four (4) parameters which constitute the design parameters. They are Ep, tp,Ga and ta. In this work we consider a 304mm x 152mm aluminum alloy plate with a 2 mm thickness, with a Young modulus of 68 GPa and a Poisson's ratio of 0.33. The plate has a crack of length a and is subjected to a tensile load of 48 GPa. In order to apply the design of experiments method to select the optimum values which give the lowest K_∞, we affected to each of the four parameters three level values. This gave us 3^4 possible combinations , hence 81 runs. Our working values were the following.

Aluminum alloy structure:	Es= 68 GPa; ts=2mm.
Composite patch:	Ep = 208 GPa (level -1); 145 GPa (level 0); 70 GPa (level 1)
	tp = 1.5 mm (level -1); 1.mm (level 0); 0.5mm (level 1)
Adhesive:	Ga= 0.9 GPa (level -1); 0.7 GPa (level 0); 0.44 GPa (level 1)
	ta= 0.2 mm (level -1); 0.15 mm (level 0) ; 0.1 mm (level 1)

We made the 81 runs that gave us the values of K_∞ for all combinations.

The results are presented in table.1. By examining this table we can observe that:

Table 1 Values of K_∞ for the different combinations of the factors.

tp	Ga	ta	Ep	K inf	Ep	K inf	Ep	Kinf
-1	-1	-1	-1	1.2320	0	1.2707	1	1.3853
-1	-1	0	-1	1.1465	0	1.1826	1	1.2891
-1	-1	1	-1	1.0360	0	1.0686	1	1.1649
-1	0	-1	-1	1.3119	0	1.3531	1	1.4751
-1	0	0	-1	1.2208	0	1.2592	1	1.3727
-1	0	1	-1	1.1031	0	1.1378	1	1.2404
-1	1	-1	-1	1.4733	0	1.5197	1	1.6566
-1	1	0	-1	1.3711	0	1.4142	1	1.5417
-1	1	1	-1	1.2389	0	1.2779	1	1.3931
0	-1	-1	-1	1.2763	0	1.3279	1	1.4741
0	-1	0	-1	1.1877	0	1.2357	1	1.3718
0	-1	1	-1	1.0732	0	1.1166	1	1.2395
0	0	-1	-1	1.3590	0	1.4140	1	1.5696
0	0	0	-1	1.2647	0	1.3159	1	1.4607
0	0	1	-1	1.1428	0	1.1890	1	1.3199
0	1	-1	-1	1.5263	0	1.5880	1	1.7628
0	1	0	-1	1.4204	0	1.4778	1	1.6405
0	1	1	-1	1.2835	0	1.3354	1	1.4824
1	-1	-1	-1	1.3871	0	1.4656	1	1.6632
1	-1	0	-1	1.2909	0	1.3639	1	1.5571
1	-1	1	-1	1.1664	0	1.2324	1	1.4070
1	0	-1	-1	1.4771	0	1.5606	1	1.7817
1	0	0	-1	1.3746	0	1.4523	1	1.6581
1	0	1	-1	1.2421	0	1.3123	1	1.4982
1	1	-1	-1	1.6589	0	1.7527	1	2.0010
1	1	0	-1	1.5438	0	1.6311	1	1.8622
1	1	1	-1	1.3949	0	1.4739	1	1.6827

The lowest values of K_∞ (desirable) are obtained with the following combinations.

K_∞= 1.0360 for tp : level -1 Ga: level -1 ta: level 1 Ep: level -1
K_∞= 1.0686 for tp : level -1 Ga: level -1 ta: level 1 Ep: level 0
K_∞= 1.0732 for tp : level 0 Ga: level -1 ta: level 1 Ep: level -1
K_∞= 1.1031 for tp : level -1 Ga: level 0 ta: level 1 Ep: level -1

The highest values of K_∞ (undesirable) are obtained with the following combinations.

K_∞= 2.0010 for tp : level 1 Ga: level 1 ta: level -1 Ep: level 1
K_∞= 1.8622 for tp : level 1 Ga: level 1 ta: level 0 Ep: level 1
K_∞= 1.7628 for tp : level 0 Ga: level 1 ta: level -1 Ep: level 1
K_∞= 1.7520 for tp : level 1 Ga: level 1 ta: level -1 Ep: level 0

We can see that the lowest values of K_∞ are obtained with the highest level of ta and the lowest levels of tp, Ga and Ep. While the highest values are obtained with the lowest level of ta and the highest levels of Ga,tp and Ep.(note that the highest level values correspond to the lowest material properties values as they are inversed in our setup).

The object of this work was to evaluate the weights of the four parameters and their interactions. For that purpose the most indicated method is the design of experiments method which is a statistical approach based upon the variance analysis (anova) of the different factors. We have chosen the full factors approach as opposed to the fractional one, because computing the 81 values of K_∞ is not a big task and a simple MATLAB program suffices. The results of this operation are presented in table.2. As we can see, each factor (parameter) has a corresponding coefficient which assesses its weight in the output (result). The interactions, represented by a product of the factors have also a corresponding weight. We could therefore model the phenomena by a polynomial of this form:

$$K_\infty = 1.3139 + (0.1066)(tp) + (0.1258)(Ga) - (0.1199)(ta) + (0.1065)(Ep) + (0.0099)(tp)(Ga) - (0.0089)(tp)(ta) + (0.0033)(tp)(Ep) - (0.0111)(Ga)(ta) + (0.0099)(Ga)(Ep) + (0.0088)(ta)(Ep) + \dots \quad (7)$$

We can easily see that the lowest value of K_∞ is obtained with the lowest levels of tp, Ga , Ep and the highest level of ta , as it can be seen on table.1 .The adhesive shear modulus, Ga has the biggest influence (0.1258) followed by adhesive thickness, ta (-0.1199), the patch young modulus Ep (0.1065) and the patch thickness tp (0.1066).The minus sign

Table 2 Results of the design of experiments analysis.

Name	Factor	Coeff	SE	T
	Const	1,3139	0,003	441,909
THR	THR (A)	0,1066	0,0012	87,837
GA	GA (B)	0,1258	0,0012	103,634
THA	THA (C)	-0,1199	0,0012	-98,749
Er	Er (D)	0,1065	0,0012	87,725
THR*GA	AB	0,0099	0,0015	6,667
THR*THA	AC	-0,0089	0,0015	-5,9644
THR*Er	AD	0,033	0,0015	22,204
GA*THA	BC	-0,0111	0,0015	-7,4555
GA*Er	BD	0,0099	0,0015	6,6521
THA*Er	CD	-0,0088	0,0015	-5,9514
THR*GA*THA	ABC	-0,0012	0,0018	-0,682
THR*GA*Er	ABD	0,0034	0,0018	1,8789
THR*THA*Er	ACD	-0,0025	0,0018	-1,3891
GA*THA*Er	BCD	-0,0012	0,0018	-0,6774
THR*GA*THA*Er	ABCD	-0,0009	0,0022	-0,3952
THR*THR	AA	0,0429	0,0021	20,394
GA*GA	BB	0,0424	0,0021	20,146
THA*THA	CC	-0,0155	0,0021	-7,3947
Er*Er	DD	0,0499	0,0021	23,74

affected to the factor ta means that the lowest value corresponds to level 1 of ta while it corresponds to the lowest levels of the other factors. Influences of the different interactions are rather small except that of (Ga.ta). It's worth noting that the first two factors are related to the adhesive, which constitutes a bridge that transfers vertically the load from the structure to the patch. The two factors related to the patch have almost the same importance. Once we have assessed the weights of the four parameters we considered their effects on the stiffness ratio and the maximum shear strain. An effective

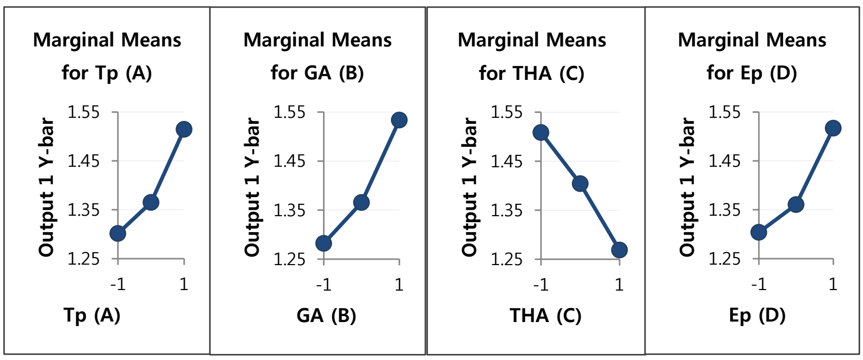

Fig. 2. Variations of the output with respect to the factors

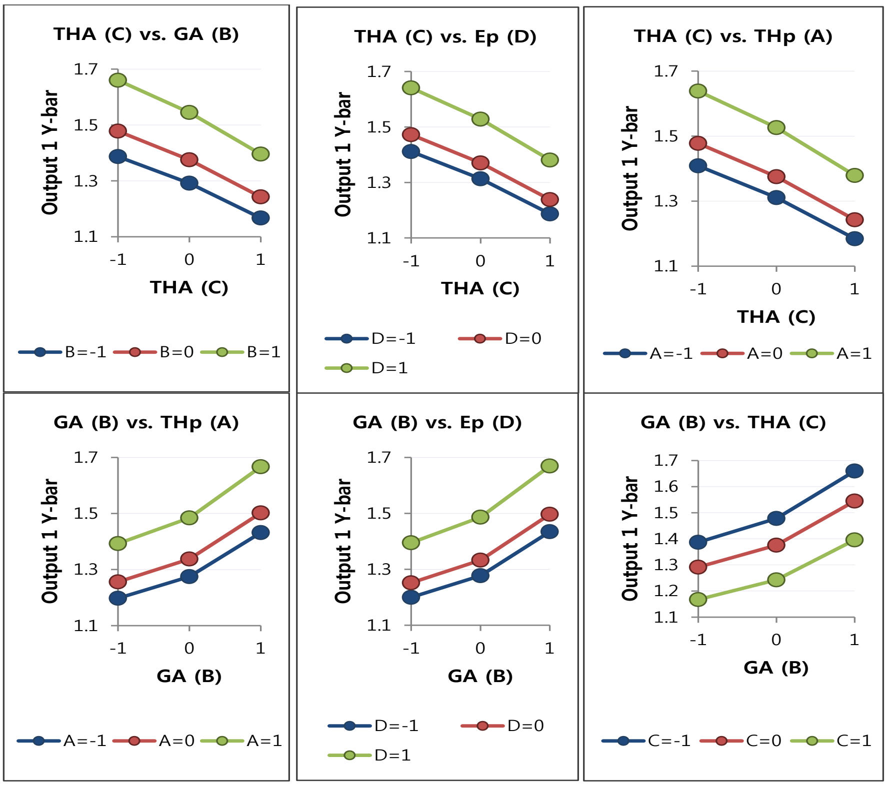

Fig. 3 a. Effects of the interactions

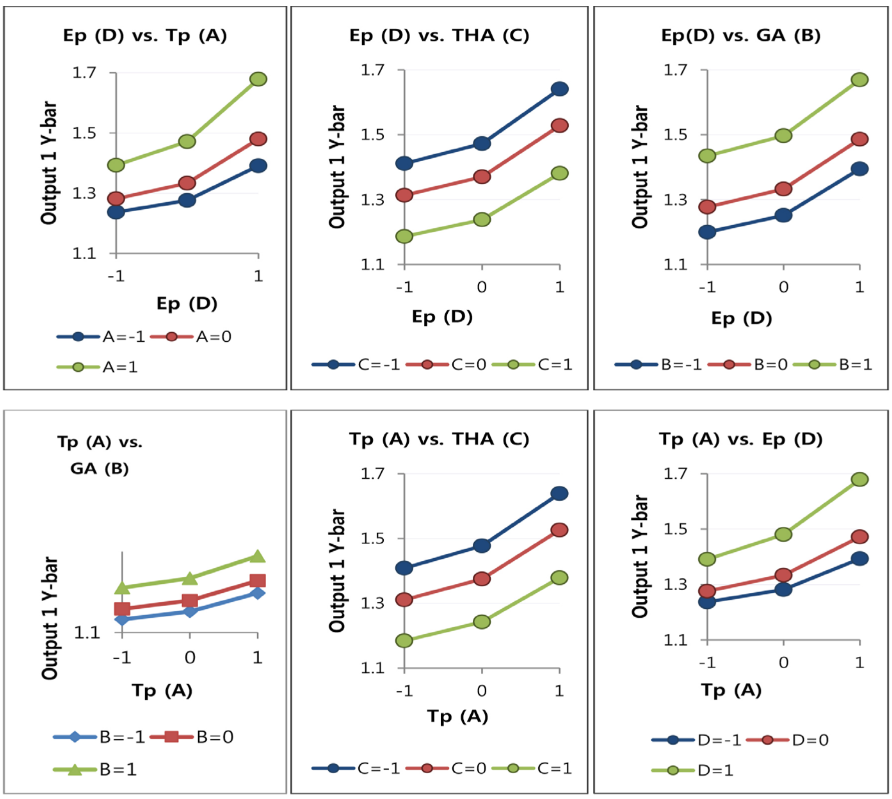

Fig. 3 b. Effects of the interactions.

design should satisfy a good load transfer by selecting Ep and tp such that their product would be close to that of Es and ts among values that allow the lowest value of K_∞. The same attention must be devoted to the shear strain in order to design a lasting repair that is not exposed to disbonding. We must select values that reduce γ_{max} among those that give the lowest K_∞. The assessments that we have obtained allowed us to obtain the optimum design by making judicious compromises and trade-offs. Figure .2 depicts how the output varies with respect to each factor and Figure.3a and 3b how it is affected by their interactions.. Then we considered how this applies to the practical example we choose. The values which give the lowest value of K_∞ correspond to, as seen earlier, level -1 for Ep, tp, Ga and level 1 for ta which corresponds to the highest values of Young modulus (208 GPa) and thickness (1.5mm) of the composite patch; the highest value of the adhesive shear modulus (0.9 GPa) and the lowest value of the adhesive thickness (0.1mm). The values of Es (68GPa) and ts (2mm) are given.

We first considered the constraint related to the stiffness ratio. The product Es*ts is equal to 136 GPa.mm and that of Ep*tp is equal to 312 GPa.mm. This gave us a transfer ratio (Eptp/Ests) equal to 2.3, which is too high compared to 1. Therefore we had to reconsider this choice. The best selection appeared to be the level 0 for both parameters, which correspond to Ep=145 GPa and tp=1mm. In deed this gave us a stiffness ratio of (145*1)/(68*2) =1.06 which is very close to 1. To validate this choice we must check with our results. By examining Fig.2 we can see that the marginal means of the factors Ep and tp are close for the level-1 and level 0.

And from Fig.3a and Fig.3b, which describes the interactions, we can see that the graphs corresponding to these two levels are not too far from each other. This comforted our choice which not only satisfies the stiffness criterion but also has two collateral advantages:

- As Ep appears in the numerator of the fraction that gives the maximum shear strain, it will lower this latter.
- The thickness tp reduces the shifting of the patch-structure assembly axis, thus reducing the non-desirable moments at the edges of the patch. Jones, R. [24].

We must maintain the highest value of Ga (level -1) as it also reduces the maximum shear strain since it appears in the denominator.

The final optimum design is obtained with the level 0 for Ep and tp, the level-1 for Ga and the level 1 for ta. The corresponding value of K_∞ (1.1166) is very satisfactory and not very far from the lowest one (1.0360).

4. CONCLUSION

The optimum design of composite patch repair is a complex task. It is impacted by four parameters (factors) that are the shear modulus of the adhesive, the module of Young of the patch and the thickness of these two parameters. With the use of the design of experiments method, we were able to assess the weight of each of these four factors and their interactions. The evaluation of the impact of each factor allowed us to identify the best combination which gave the lowest K_∞ and ensured the satisfaction of constraints relative to the transfer of load and the minimization of shear strain.

REFERENCES

1. Wang, C. H., “Introduction to Fracture Mechanics. Airframes and Engines Division”. Aeronautical and Maritime Research Laboratory Published by DSTO Aeronautical and Maritime Research Laboratory, July 1996, 10-13.
2. Anderson, T. L., “Fracture mechanics; Fundamentals and Applications”. Taylor & Francis Group 6000 Broken Sound Parkway NW, Suite 300Boca Raton, FL 33487-2742 by Taylor & Francis Group, LLC, 2005, 29-39.
3. Griffith, A. A., “The Phenomena of Rupture and Flow in Solids”. Philosophical Transactions, Series A, 1920, 221, 163–198.
4. Irwin, G. R., ‘‘Onset of Fast Crack Propagation in High Strength Steel and Aluminum Alloys.” Sagamore Research Conference Proceedings, 1956, 2, 289–305.
5. Hassani, A. and Ravaee, R., “Characterization of transverse crack and crack growth in tansverse railway rail”. Iranian Journal of Materials Science and Engineering., 2008, 5 (2) , 22-31.
6. Kotoussov, A. and Chang, D., “Local Plastic Collapse Conditions for a Plate Weakened by

Two Closely Spaced Collinear Cracks". Engineering Fracture Mechanics., 2014, 127, 1-11.

7. Chang, D. and Kotoussov, A ., "A fatigue crack growth model for interacting cracks in a plate of arbitrary thickness". Fatigue and Fracture of Engineering Materials and Structures., 2014, 37, 1254-1267.
8. Haddad Sabzevar, M. and Fredriksson, H., "Hot crack formation in pure Cu and Cu- 30% Zn alloy During in Situ Solidification". Iranian Journal of materials Science and Engineering., 2006, 3(1), 9-15.
9. Ding, J., Wang, L. S., Song, K., liu, B. and Huang, X., "Molecular Dynamics Simulations of crack Propagation in a Single-Cristal Aluminum Plate with Central Cracks". Journal of nanomaterials., 2017, 1-12.
10. Baker A. A.,"Repair of cracked or Defective Metallic Aircraft Components with Advanced Fibre". Elsevier Applied Sciences Publishers Ltd., 1984, 153-181.
11. Baker A. A., "Fiber Composites Repairs of Cracked Metallic Aircraft Components-practical and Basic Aspects". Composites. , 1987, 18(4), 283-308.
12. Jones R., Davis, M., Callinan, R. J. and Mallinson, G. D., "Crack Patching: Analysis and Design". Journal of Structural Mechanics., 1982, 10, 177-190.
13. Barthlomeusz, R. A., Paul, J. J. and Roberts, J. D., "Application of Bonded Composite Repair technology to Civil Aircraft". Aircraft Engineering, 1993, 65, 4-7.
14. Rose, L. R. F., "Theoretical Analysis of Crack Patching. Bonded Repair of Aircraft Structures". ed. A. A. Baker and R. Jones. Martinus Nijhof T Publishers. Dordrecht, 1988, 49-76.
15. Chue, C. H., Lin, L. A., and Wang, S. C., "Optimum Selection of Ply Orientation in Bonded Repair under biaxial Loading". Engineering Fracture Mechanics., 1994, 48, 91-101.
16. Brighenti, R., Carpinteri, S. and Vantadori, A., "Genetic algorithm applied to optimisation of patch repairs for cracked plates". Computer methods in applied mechanics and engineering., 2006, 196, 466-475.
17. Tsamasphyros, G. J., Kanderakis ,G. N., Furnarakis, N. K. and Marioli-Riga, Z. P., "Detection of patch debonding in composite repaired cracked metallic specimens, using optical fibers and sensors" . Proceedings of SPIE - The international Society for Optical Engineering, 2003.
18. Ait Yala, A. and Megueni, A., "Optimisation of Composite Patches Repairs with the Design of experiments Method". Materials and Design., 2009, 30, 200-205.
19. Benoist, D. Y., Tourbier, S. and Germain, P. W., "Experimental design: construction and analyses". Doc. Lavoisier, Paris, 1994, 12- 46.
20. Fischer R. A., "Statistical methods for research workers". Oliver and Boyd ed. London, 1925, 241-271.
21. Taguchi, G. and Konishi, S., "Orthogonal Arrays and Linear Graphs: tools for Quality Engineering". Allan Park, MI: ASI, 1987, 35-38.
22. Jones, R., "Neutra1 Axis Offset Effects due to Crack Patching"., Composite Structures., 1983, 1, 163-174.