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PREDICTION OF NATURAL FREQUENCIES FOR TRUSS STRUCTURES WITH UNCERTAINTY USING THE SUPPORT VECTOR MACHINE AND MONTE CARLO SIMULATION

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ABSTRACT

In this study, the support vector machine and Monte Carlo simulation are applied to predict natural frequencies of truss structures with uncertainties. Material and geometrical properties (e.g., elasticity modulus and cross-section area) of the structure are assumed to be random variables. Thus, the effects of multiple random variables on natural frequencies are investigated. Monte Carlo simulation is used for probabilistic eigenvalue analysis of the structure. In order to reduce the computational cost of Monte Carlo simulation, a support vector machine model is trained to predict the required natural frequencies of the structure computed in the simulations. The provided examples demonstrate the computational efficiency and accuracy of the proposed method compared to the direct Monte Carlo simulation in the computation of the natural frequencies for trusses with random parameters.

Keywords: Machine learning; support vector machine; truss; random eigenvalue problem; uncertainty quantification; Monte Carlo simulation.

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1. INTRODUCTION

Uncertainties are effective in the analysis and design procedures of a structural system, and hence the use of a probabilistic approach is necessary. Therefore, various methods have been developed for uncertainty quantification of structures [1-4]. One of the well-known methods for probabilistic analysis is Monte Carlo simulation (MCS), known as a sampling method,

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which employs many realizations based on randomly generated sampling sets for uncertain variables. The MCS has been used in various reliability analyses [5, 6]. However, although the MCS is a robust method for probabilistic structural analysis, it needs a high computational cost as each simulation corresponds to a time-consuming structural analysis. To overcome this deficiency, a suitable sampling method or surrogate model (like a machine learning-based model) can reduce the computational efforts. For example, the importance sampling approach is utilized to reduce the total number of samples. In contrast, a machine learning tool can be used as a surrogate model instead of the actual model.

Numerous applications of machine learning have emerged with increasing advances in computer technology. Nowadays, machine learning methods are widely used in many engineering fields, such as computational mechanics [7, 8], structural materials [9, 10], structural optimization [11, 12], face recognition [13], spam detection [14], error-resilient architectures [15, 16], and electric power systems [17]. Support vector machine (SVM) is a machine learning tool applied to both classification and regression problems, which was first identified by Vapnik et al. [18]. The applications related to structural mechanics mainly include the following subjects: structural materials, earthquake engineering, wind engineering, and structural health monitoring [9, 11, 19-21]. More recently, the MCS and SVM were employed to calculate the failure probability of the structure [5].

Uncertainty representation of natural frequencies is crucial for studying the dynamic characteristics of structural systems [22]. Stochastic methods have widely been developed to consider uncertainties when sufficient statistical information is available. In these methods, uncertainties are modeled as random variables, processes, or fields. Uncertainty characterization is represented by the probability density function, mean value, variance, etc. Nevertheless, if the information for the probability density function is unavailable, the interval of upper and lower bounds of random variables can be employed to represent uncertainty.

Natural frequencies of a structure with random parameters are often obtained by solving the random eigenvalue problems. Scheidt and Purker [23] investigated the random eigenvalue problems. Various approaches [22, 24] were employed to solve these problems, such as direct MCS method [24], and perturbation method [22]. Hollot and Bartlett [25] carried out an study on eigenvalues of interval matrices. Chen et al. [26] presented the perturbation methods for calculating the bounds of eigenvalues of vibration systems with interval parameters. Qiu et al. [27] computed the eigenvalue bounds of structures with uncertain-but-bounded parameters using vertex theorem. Gao [28] presented the interval factor technique for interval analysis of natural frequency and mode shape of structures with interval parameters. Modares et al. [29] proposed an element-by-element formulation to consider interval eigenvalue problem. Angeli et al. [30] studied the natural frequency intervals for the systems having polytopic uncertainty. Many studies on stochastic problems were carried out to determine dynamic characteristics of structures considering an uncertain model. However, in real-world problems, usually a large number of variables and design parameters exist in a structural system. Since some may have sufficient statistical information, stochastic and interval models are needed simultaneously. In this regard, various attempts have been made on solving the mixed stochastic problems in either static [31, 32] or dynamic [33] case.

In this study, the MCS is utilized for calculating the natural frequencies of truss structures

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with uncertainties. Also, various random variables are considered in the MCS to calculate probabilistic eigenvalue analysis of the structure. To accelerate the MCS, the SVM is employed to predict the natural frequencies of the structure such that each simulation in the MCS can be performed faster. The computational efficiency and accuracy of the proposed method are compared to those of the direct MCS using three examples.

The remainder of this article is presented as follows. Section 2 focuses on the research background of the SVM. In Section 3, a review of eigenvalue analysis for determining the natural frequencies of truss structures is given, and then the proposed probabilistic approach is discussed. Section 4 provides the illustrative examples. Finally, the conclusions are summarized in Section 5.

2. SUPPORT VECTOR MACHINE

The SVM was invented in the early 1960s, and became popular in the 1990s [18, 34]. The SVM is a machine learning tool for regression analysis and data classification [8, 9]. To classify data into groups of samples, the SVM finds a hyperplane (decision boundary) in a k-dimensional space when considering k features. Clearly, a two-dimensional space can be separated by a line, but a higher-dimensional space can be separated by a hyperplane. Such a plane has maximum distance to the groups of samples. As shown in Figure 1, the points in close proximity to the hyperplane are referred to as *support vectors*, which can be dramatically effective in the position and orientation of the hyperplane. The SVM utilizes a particular form of mathematical functions defined as kernels transforming inputs into the required form. Since the SVM relies on kernel function, it is a nonparametric technique. The kernel function can be in linear or nonlinear form. An optimization problem is defined to find the optimal hyperplane, which can be solved by optimization techniques. Lagrange multipliers are utilized to get this problem into a form that can be solved analytically.

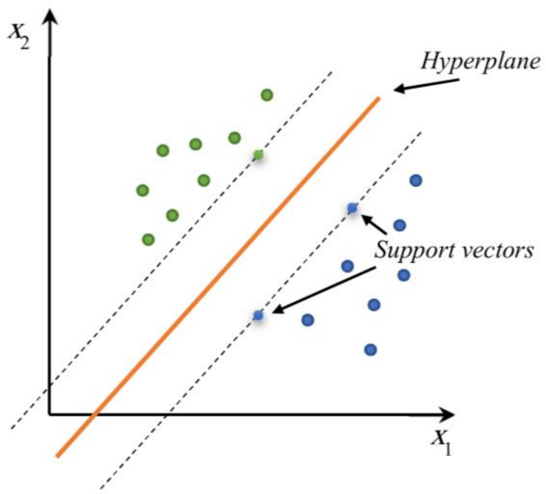

Figure 1. Schematic of a two-class SVM

In this study, the SVM implemented in MATLAB for regression analysis is utilized,

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which is the linear epsilon-insensitive SVM regression. In ε-SVM regression, the set of training data consists of a multivariate set of N observations ($\mathbf{x}_n$) and observed response values (y_n). As mentioned before, the goal is to find a function $f(\mathbf{x})$ deviating from y_n by a value no greater than ε for each training point $\mathbf{x}$, and at the same time is as flat as possible.

In the primal formula of linear SVM regression, the following function is sought

$$f(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{x} + b \tag{1}$$

such that it is as flat as possible. In order to find $f(x)$ with the minimal norm value, the following convex optimization problem is minimized

$$J(\mathbf{w}) = \frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w} \tag{2}$$

with all residuals limited to an upper bound ε:

$$\forall n : \left| y_n - (\mathbf{w}^{\mathrm{T}}\mathbf{x} + b) \right| \le \varepsilon \tag{3}$$

There is a possibility of finding no function like $f(x)$ to satisfy these constraints for all points. This issue is resolved by introducing slack variables ξ_n and ξ_n^* for each point. These slack variables allow regression error values up to ξ_n and ξ_n^*, while satisfying the required conditions. Such a technique is similar to the concept of *soft margin* in the SVM classification.

The primal formula is determined by adding the slack variables to Eq. (2), resulting in the following objective function:

$$J(\mathbf{w}) = \frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w} + C\sum_{n=1}^{N}(\xi_n + \xi_n^*) \tag{4}$$

with

$$\begin{aligned} &\forall n : y_n - (\mathbf{w}^{\mathrm{T}}\mathbf{x} + b) \le \varepsilon + \xi_n \\ &\forall n : (\mathbf{w}^{\mathrm{T}}\mathbf{x} + b) - y_n \le \varepsilon + \xi_n^* \\ &\forall n : \xi_n \ge 0 \\ &\forall n : \xi_n^* \ge 0 \end{aligned} \tag{5}$$

where C is a positive numerical value controlling the penalty applied to observations that fall outside the epsilon margin (ε), and leads to avoid overfitting (regularization). This value

[DOI: 10.22068/ijoce.2024.14.2.583]

provides a trade-off between the flatness of $f(x)$ and the amount up to which deviations greater than ε are tolerated.

The loss function is linear ε-insensitive and ignores errors when the distance between the predicted values and observed values is less than ε. The loss function is expressed in terms of the distance between the ε boundary and observed value y, that is

$$L_{\varepsilon}=\begin{cases}0 & \text{if } \left|y-f(\xi)\right|\le\varepsilon \\ \left|y-f(\xi)\right|-\varepsilon & \text{Otherwise}\end{cases} \tag{6}$$

Lagrange dual formulation of the optimization problem defined in Eq. (4) is computationally more efficient to be solved. The solution of the dual problem gives a lower bound to the solution of the primal problem. The optimal values of the primal and dual problems are not necessarily equal, and the difference is known as *duality gap*. Nevertheless, the value of the optimal solution to the primal problem is provided by the solution of the dual problem when the problem is convex and satisfies a constraint qualification condition.

In order to determine the dual formula, the Lagrangian from of the primal function is constructed by defining nonnegative multipliers α_n and α_n^* for each observation $\mathbf{x}_n$. Hence, one can minimize

$$L(\alpha)=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)\mathbf{x}_i^{\mathrm{T}}\mathbf{x}_j+\varepsilon\sum_{i=1}^{N}(\alpha_i+\alpha_i^*)+\sum_{i=1}^{N}y_i(\alpha_i^*-\alpha_i) \tag{7}$$

subjected to the following constraints

$$\begin{gathered}\sum_{n=1}^{N}(\alpha_n-\alpha_n^*)=0\\ \forall n: 0\le\alpha_n<C\\ \forall n: 0\le\alpha_n^*<C\end{gathered} \tag{8}$$

The parameter $\mathbf{w}$ can be completely expressed as a linear combination of the training observations, as given by

$$\mathbf{w}=\sum_{n=1}^{N}(\alpha_n-\alpha_n^*)\mathbf{x}_n \tag{9}$$

The following function relies on the support vectors and predicts new values:

$$f(\mathbf{x})=\sum_{n=1}^{N}(\alpha_n-\alpha_n^*)\mathbf{x}_n^{\mathrm{T}}\mathbf{x}+b \tag{10}$$

To find optimal solutions, the Karush-Kuhn-Tucker (KKT) conditions for linear SVM

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regression are used as the optimization constraints defined by

$$\begin{aligned}
&\forall n : \alpha_n(\varepsilon + \xi_n - y_n + \mathbf{w}^{\mathrm{T}}\mathbf{x}_n + b) = 0 \\
&\forall n : \alpha_n^*(\varepsilon + \xi_n^* + y_n - \mathbf{w}^{\mathrm{T}}\mathbf{x}_n - b) = 0 \\
&\forall n : \xi_n(C - \alpha_n) = 0 \\
&\forall n : \xi_n^*(C - \alpha_n^*) = 0
\end{aligned} \tag{11}$$

which imply that all observations exactly inside the epsilon tube have Lagrange multipliers α_n and α_n^* being equal to zero. When either α_n or α_n^* is not zero, the corresponding observation is referred to as a *support vector*.

To establish a nonlinear SVM regression model, the product $\mathbf{x}_i^{\mathrm{T}}\mathbf{x}_j$ is replaced by kernel function $G(\mathbf{x}_i, \mathbf{x}_j)$ because $\mathbf{x}_i^{\mathrm{T}}\mathbf{x}_j$ is also known as a linear kernel function. A nonlinear kernel function can usually be expressed in Gaussian or polynomial form. However, the linear SVM model is employed in this paper.

Since the aforementioned minimization problem can be in standard quadratic programming form, it is solved by common programming approaches which can be computationally expensive. Using a *decomposition method* can accelerate the computation and prevent running out of memory. However, the *sequential minimal optimization* is the most popular technique for solving the SVM problems because the Lagrange multipliers used in this technique are solved analytically [35]. Each solver needs a convergence criterion, and there are several convergence criteria for the solvers used for the SVM, such as *feasibility gap*, *gradient difference*, and *largest KKT violation*.

3. PROBABILISTIC EIGENVALUE ANALYSIS

Here, the analysis of vibration frequencies for truss structures is reviewed. Then, the probabilistic analysis using the MCS and SVM is discussed.

3.1. Vibration frequencies of truss structures

In order to compute the natural frequencies of a structure, an eigenproblem should be solved by incorporating the stiffness and mass matrices of the structure [36, 37]. The stiffness matrix of a three-dimensional truss element is presented as follows:

$$\mathbf{k}^e = \frac{EA}{L}\begin{bmatrix}
C_x^2 & C_xC_y & C_xC_z & -C_x^2 & -C_xC_y & -C_xC_z \\
 & C_y^2 & C_yC_z & -C_xC_y & -C_y^2 & -C_yC_z \\
 & & C_z^2 & -C_xC_z & -C_yC_z & -C_z^2 \\
 & & & C_x^2 & C_xC_y & C_xC_z \\
 & & & & C_y^2 & C_yC_z \\
\mathit{Symmetric} & & & & & C_z^2
\end{bmatrix} \tag{12}$$

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where

$$C_x = \frac{x_j - x_i}{L}, \quad C_y = \frac{y_j - y_i}{L}, \quad C_z = \frac{z_j - z_i}{L} \tag{13}$$

where $\mathbf{k}^e$ represents the stiffness matrix of a truss element formed by connecting the nodes i and j; also, L, A and E are the element length, cross-section area, and elasticity modulus, respectively; x_i , y_i and z_i denote the Cartesian coordinates of the ith node.

Also, the consistent mass matrix of a three-dimensional truss element is expressed by

$$\mathbf{m}^e = \frac{\rho AL}{6}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix} \tag{14}$$

in which ρ is the density. For briefness, the mass and stiffness matrices of the two-dimensional truss element are not mentioned here.

The following eigenproblem can be established by assembling all mass and stiffness elements:

$$\mathbf{K}\phi_k = \omega_k^2 \mathbf{M}\phi_k \tag{15}$$

where **M** and **K** show the mass and stiffness matrices of the truss structure. Also, ω_k and ϕ_k are the kth circular frequency and mode shape vector.

3.2. Probabilistic eigenvalue analysis using the MCS-SVM

The MCS is a kind of computational algorithm that employs repeated random sampling for estimating the possible outcomes of an uncertain event to obtain the statistical measures for a range of results. In other words, the MCS builds a model of possible outcomes by leveraging a probability distribution for any variable having inherent uncertainty. Then, it recalculates the results repeatedly, each time using a different set of random numbers. The MCS involves three fundamental steps:

(i) The predictive model is built by identifying both the independent variables (the input or predictor variables) and the dependent variable to be predicted.

(ii) Probability distributions of the independent variables are specified. In this regard, historical data and/or the analyst's subjective judgment may be used. Then, the random values of the independent variables are generated using the probability distributions.

(iii) The simulations are repeatedly performed considering the predefined number of

[DOI: 10.22068/ijoce.2024.14.2.583]

samples, such that each of which corresponds to a realization of the random variable(s) followed by the evaluation of the independent variables. For example, a deterministic eigenvalue problem is solved for each realization. Finally, the procedure is terminated upon reaching the desired accuracy of the response statistics (e.g., mean and standard deviation values).

Although the MCS is a simple and robust method for implementing the response variability calculation in the framework of stochastic structural mechanics, the estimation accuracy of the MCS depends on the number of samples. For example, the standard deviation estimate is inversely proportional to the square root of the number of samples [38]. Therefore, the solution of many deterministic problems corresponding to many samples needs a remarkable computational cost, particularly for a large-scale system with considerable stochastic dimension.

In this study, a surrogate model is formed by the SVM regression, instead of the eigenvalue analysis requiring a structural model, to speed up the MCS. The MCS-SVM is carried out according to the following steps:

Step 1: a data set of observations containing the input random variables (features) and the resulting natural frequencies (response values) is created. The *n*th training point corresponds to $\mathbf{x}_n$ and y_n taken as some random structural parameters and natural frequency of the structure, respectively.

Step 2: the SVM regression model is trained using the created data set.

Step 3: the MCS is utilized for the solution of probabilistic eigenvalue analysis but no matrix structural analysis for the eigenvalue analysis is performed in each simulation. Instead, each simulation uses the trained SVM model to predict the natural frequencies of the structure, thereby reducing the computational cost.

4. ILLUSTRATIVE EXAMPLES

Here, three examples are provided to demonstrate the efficiency of the proposed MCS-SVM method. Both MCS and MCS-SVM methods are employed for the examples to compare their solution accuracy and computational cost. Obviously, the MCS uses an eigenvalue analysis for each simulation, but the MCS-SVM uses the SVM for each simulation instead. The total number of Monte Carlo simulations is selected as 20,000. For the SVM, the generated data with a size of N=1000 are randomly split into two sets: the training set consists of 90 percent of data, and the testing set includes 10 percent of data. These 1000 data points are obtained using eigenvalue analyses of the structure corresponding to various realizations of random variables. In order to consider more variety of data, here the defined standard deviation of every normal random variable is multiplied by 2 when generating the data set, while that of every uniform random variable is multiplied by 1.5.

For each testing set, the mean square error is calculated by

$$MSE = \frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y}_j)^2 \tag{16}$$

[DOI: 10.22068/ijoce.2024.14.2.583]

where y_j and $\hat{y}_j$ are the observed (true) and predicted output values corresponding to the jth data point; n is the number of data points utilized for testing. All the analyses are performed using a laptop with the Intel Core i7-7700HQ CPU @ 2.80 GHz.

4.1. An 8-bar planar truss

The first example is an 8-bar truss made of aluminum, as illustrated in Figure 2 where the nodal and member numberings of the truss are also shown. This problem was previously solved in Ref. [39]. The mean values of the elasticity modulus, mass density and cross-sectional area are 7.1008×10^4 MPa, 2.8497×10^3 kg/m^3, and 4.8×10^{-4} m^2, respectively. In order to investigate the effects of uncertainties, different coefficients of variation are defined for uniformly distributed random variables including cross-sectional area, member length, density, and elasticity modulus. The coefficients of variations for cross-sectional areas and member lengths of all the elements are taken as 0.1. The same value is also taken for the coefficients of variations of elasticity modulus and density of elements 1 to 6, but those of elasticity modulus and density of elements 7 and 8 are selected as 0.13.

Considering the input random variables used, the data set of the SVM model is created within 1.3925 s using 6 features for each data point. The number of random variables used for elasticity modulus, mass density, cross-sectional area, and member length are 2, 2, 1, and 1, respectively. The observed and predicted values of the fundamental frequency are illustrated in Figure 3. Mean values of the fundamental frequency obtained with the MCS and MCS-SVM are 76.6377 Hz and 77.7309 Hz, respectively. Also, the standard deviation values of the fundamental frequency obtained with the MCS and MCS-SVM are 9.3411 Hz and 9.3029 Hz, respectively. The computational times of MCS and MCS-SVM are equal to 15.1566 s and 0.8995 s, respectively. It should be noted that the computational time of the MCS-SVM reported in this study includes the training time of the SVM model and the analysis time of the MCS. The results show that the MCS-SVM performs faster than the MCS, providing appropriate accuracy. Based on Eq. (16), the mean square error equals 3.9205. Since the standard deviation used for creating the data set is larger than that of the samples used for the MCS-SVM to increase the variety of data, the error is not very small.

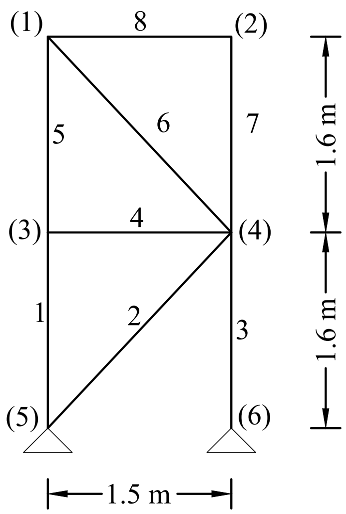

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Figure 2. An 8-bar planar truss.

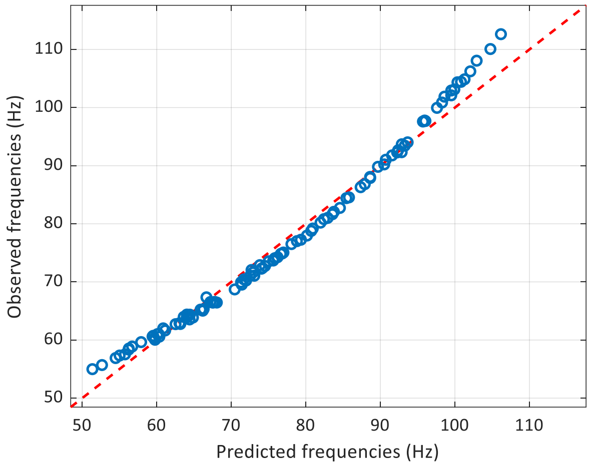

Figure 3. The observed and predicted values of the fundamental frequency for the 8-bar truss.

4.2. A 72-bar space truss

A 72-bar space truss is shown in Figure 4 representing the nodal and member numberings. The mean value of elastic modulus is selected as 6.98×10^4 MPa, while the density is taken as 2770 kg/m^3. A lumped mass of 2270 kg is also placed on to the top nodes of truss. The cross-sectional areas of members are classified into 16 groups: (1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30}, (7) A_{31}–A_{34}, (8) A_{35}–A_{36}, (9) A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}–A_{58}, (14) A_{59}–A_{66}, (15) A_{67}–A_{70}, (16) A_{71}–A_{72}. The mean cross-sectional area for each group is equally selected as 10^{-3} m^2. The coefficients of variations for elasticity modulus and cross-sectional areas of all members are assumed to be 0.04 and 0.05. The elasticity modulus and cross-sectional area are random variables with normal distribution, respectively. The same random elasticity modulus is assigned to all members, but the random cross-sectional area for all members of each group is identical. However, the other structural parameters are assumed to be deterministic.

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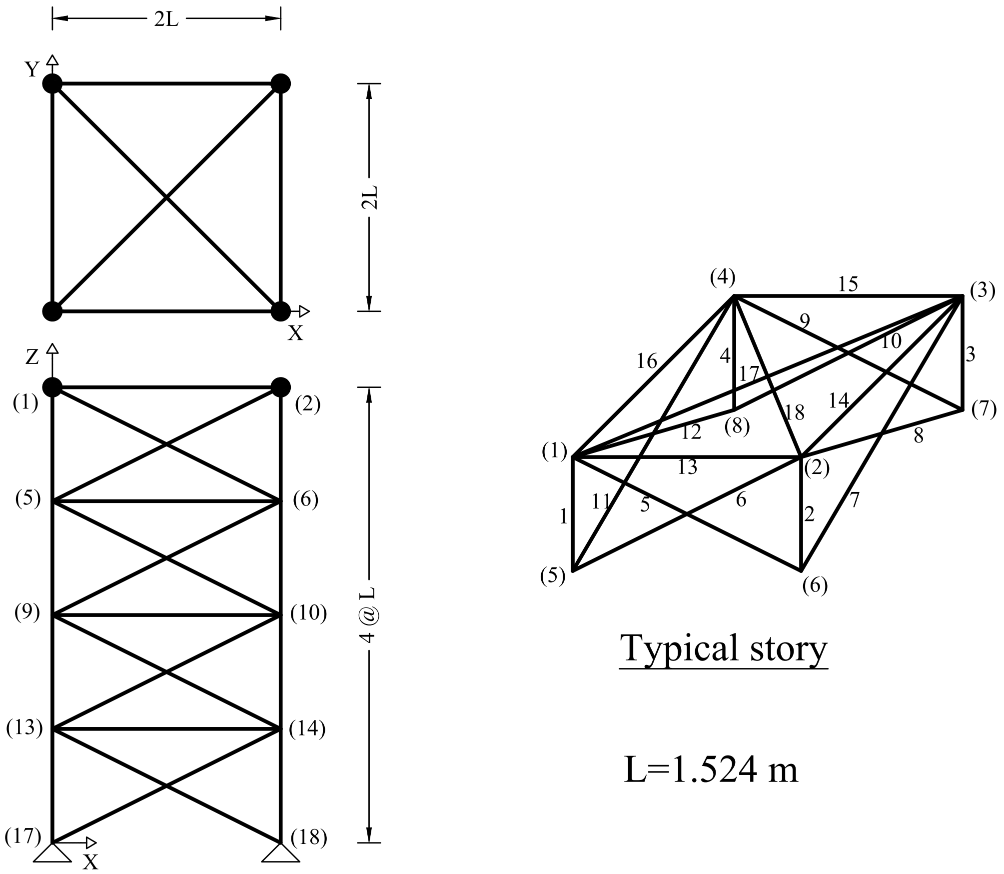

Figure 4. A 72-bar truss; lateral view, top view, and a typical story of the structure are shown.

Since three natural frequencies are selected as outputs, three SVM models are constructed. The data set of the SVM models is created within 6.9435 s using 17 features for each data point. The number of random variables for the cross-sectional area and elasticity modulus are 16 and 1, respectively. The observed and predicted values of the natural frequencies of the first three vibration modes are shown in Figure 5. Since frequencies of the first and second modes are close together, Figures 5a are 5b are very similar. Mean and standard deviation values of the natural frequencies obtained with the MCS and MCS-SVM are reported in Table 1 which also lists the computational times of MCS and MCS-SVM. As observed in Table 1, the MCS-SVM outperforms the MCS when comparing the elapsed times, while giving a desirable solution accuracy. Furthermore, the mean square errors of three SVM models are reported in Table 2.

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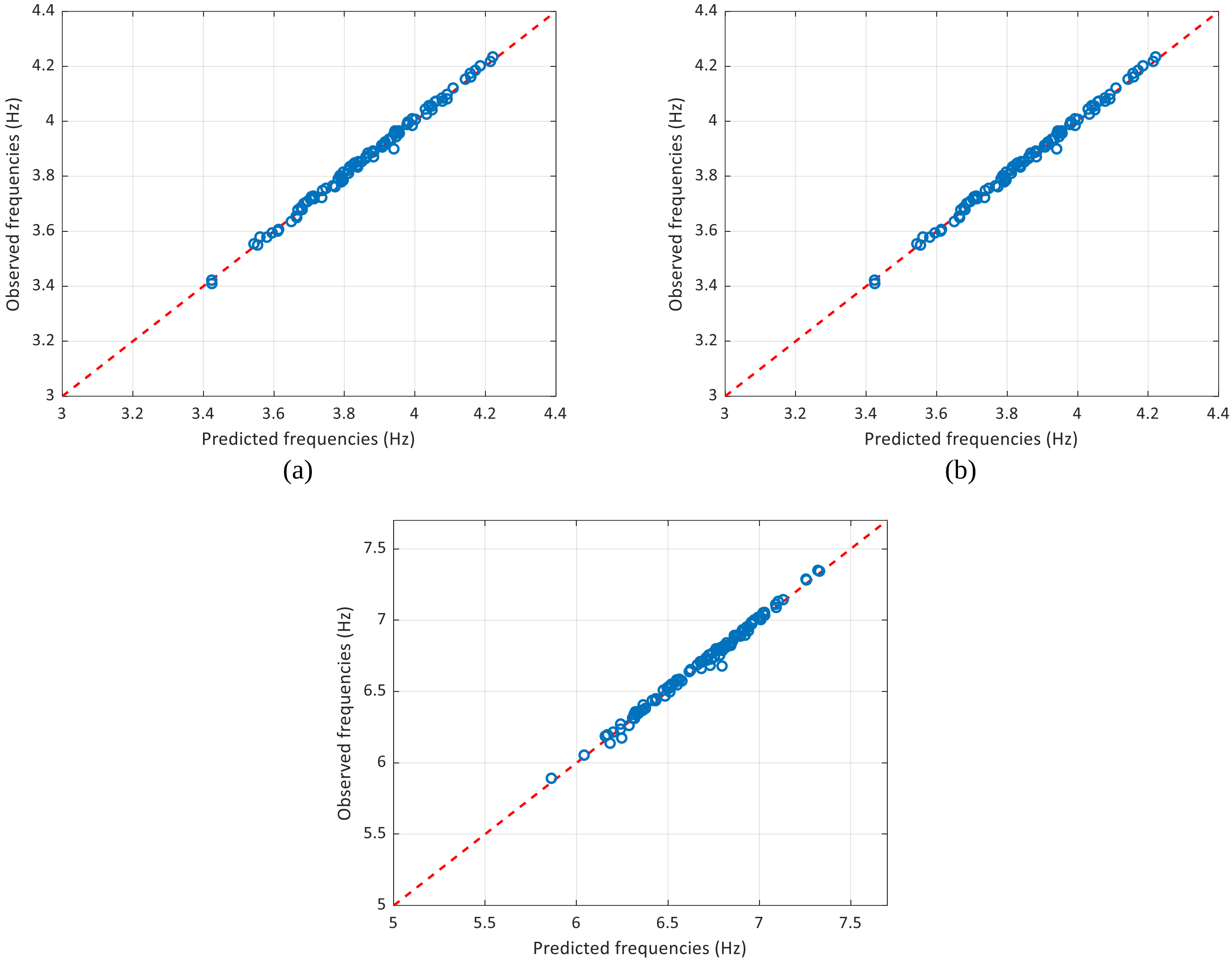

Figure 5. The observed and predicted values of natural frequencies for initial vibration modes of the 72-bar truss: (a) the first mode, (b) the second mode, and (c) the third mode.

Table 1. Mean and standard deviation values of the natural frequencies of the 72-bar truss obtained with the MCS and MCS-SVM.

Natural frequency (Hz)	MCS		MCS-SVM	
	Mean	Standard deviation	Mean	Standard deviation
f_1	3.8775	0.0892	3.8589	0.0887
f_2	3.8775	0.0892	3.8589	0.0887
f_3	6.6987	0.1582	6.6673	0.1580
Elapsed time (s)	120.3169		10.6980	

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Table 2. The mean square errors for the SVM models corresponding to different natural frequencies of the 72-bar truss.

MSE		
f_1	f_2	f_3
0.000138	0.000138	0.000692

4.3. A 600-bar truss dome

A single-layer truss dome described in [40] is illustrated in Figure 6. The span length of the dome is equal to 28 m, while its height is 7.5 m. This truss structure has 216 nodes and 600 members generated by cyclic replicating a substructure with 9 nodes and 25 members. The angle between every two subsequent substructures is 15 degrees, resulting in 24 similar substructures. The mean value of cross-sectional area, elasticity modules and density for all members are 2×10^{-3} m^2, 2×10^5 MPa and 7850 kg/m^3, respectively. All the ground-level nodes are simply supported. Mass density, elasticity modulus, and cross-sectional area are random variables with normal distribution and coefficient of variations 0.1, 0.1, and 0.16, respectively. These random variables are identical for all members.

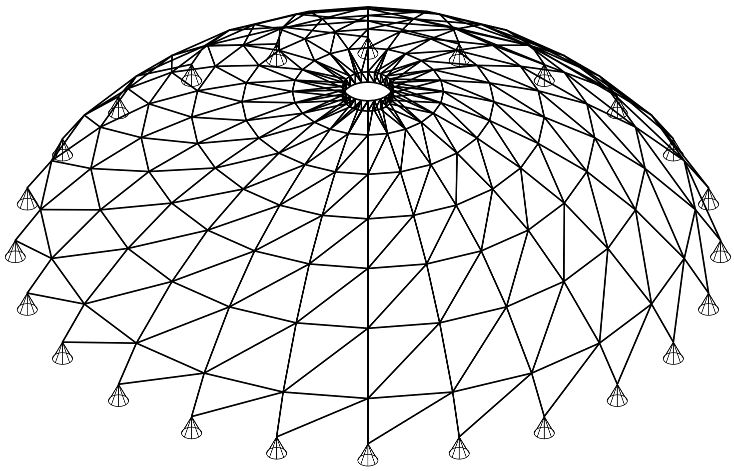
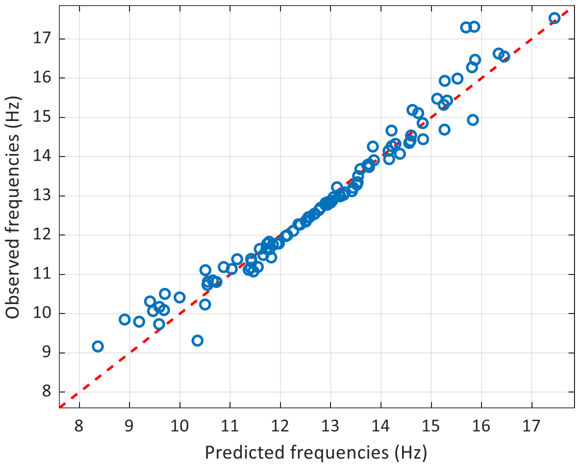
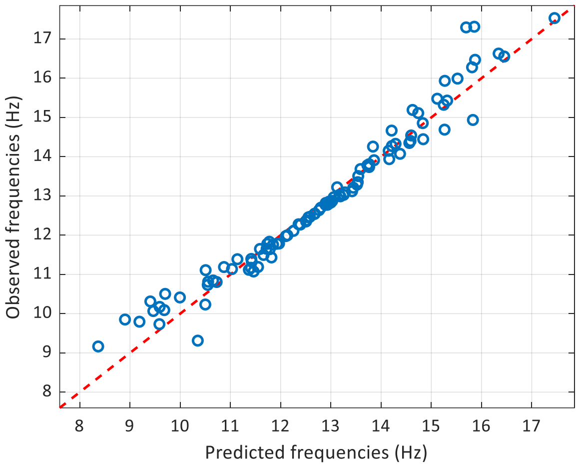


Figure 6. A 600-bar truss dome.

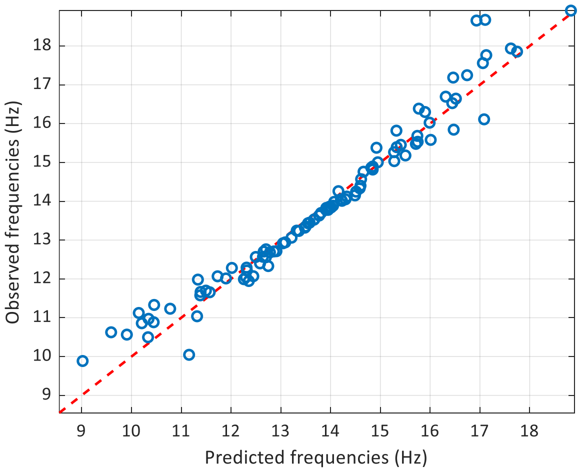
Here six natural frequencies are needed, and hence six SVM models are constructed. The data set is created within 68.9940 s, for which 3 features comprising cross-sectional area, elasticity modulus and mass density are considered for each data point. The observed and predicted values of the first six natural frequencies are indicated in Figure 7. Mean and standard deviation values of the natural frequencies computed with the MCS and MCS-SVM are listed in Table 3 which also reports the computational times. According to the results, the MCS-SVM is much faster than the MCS. Also, mean and standard deviation values obtained with the MCS-SVM are very close to those obtained with the MCS. The mean square errors given in Table 4 are relatively small, but the errors become larger for predicting frequencies of higher modes. The reason can be the requirement for more data points to reach a better accuracy.



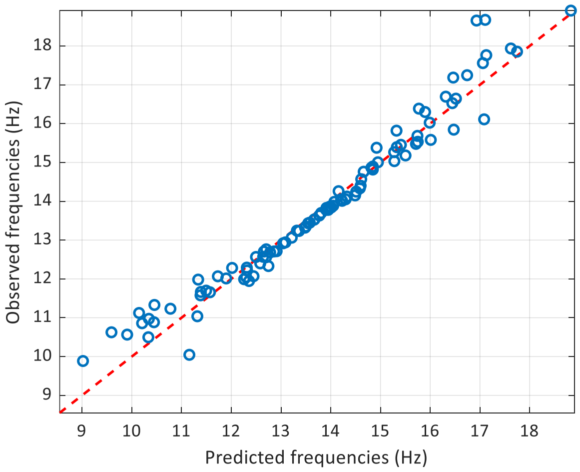
(a)



(b)



(c)



(d)

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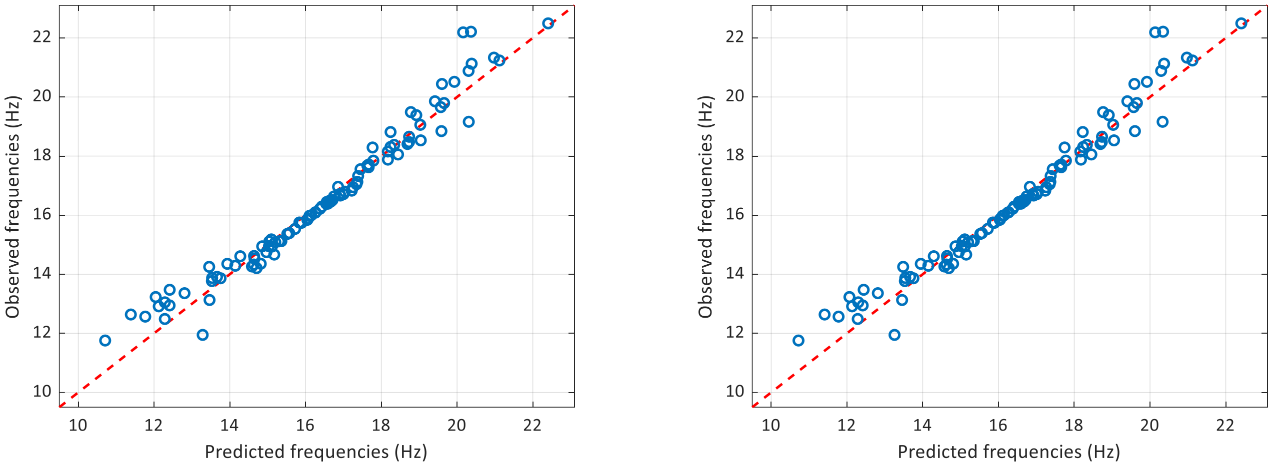

(e) (f)

Figure 7. The observed and predicted values of natural frequencies for initial vibration modes of the 600-bar truss dome: (a) the first mode, (b) the second mode, (c) the third mode, (d) the fourth mode, (e) the fifth mode, and (f) the sixth mode.

Table 3. Mean and standard deviation values of the natural frequencies of the 600-bar truss dome obtained with the MCS and MCS-SVM.

Natural frequency (Hz)	MCS		MCS-SVM	
	Mean	Standard deviation	Mean	Standard deviation
f_1	12.5628	0.8993	12.6574	0.9058
f_2	12.5628	0.8993	12.6574	0.9058
f_3	13.5502	0.9700	13.6523	0.9783
f_4	13.5502	0.9700	13.6523	0.9783
f_5	16.1168	1.1538	16.2401	1.1664
f_6	16.1168	1.1538	16.2375	1.1653
Elapsed time (s)	1.3332×10^3		2.0777	

Table 4. The mean square errors for the SVM models corresponding to different natural frequencies of the 600-bar truss dome.

MSE					
f_1	f_2	f_3	f_4	f_5	f_6
0.162215	0.162215	0.189055	0.189055	0.267757	0.267541

5. CONCLUSIONS

In this paper, the MCS with SVM is utilized to predict the natural frequencies of truss structures with uncertain parameters. These uncertain parameters include elasticity modulus, mass density, cross-sectional area, and, or member length of the structure, which are

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assumed to be random variables with uniform or normal distribution. The resulting random eigenvalue problem is solved with the MCS to calculate the natural frequencies of the structure with random parameters. However, due to the high computational cost of the MCS when performing eigenvalue analyses, here the proposed SVM model predicts the required eigenvalues for each simulation in the MCS. Therefore, an SVM-based surrogate model is trained with fewer data points than the samples (simulations) used in the MCS to predict the natural frequencies efficiently. Three trusses, from small- to large-scale size, with different random parameters are employed as numerical examples to demonstrate the capabilities of the proposed MCS-SVM in comparison with the MCS. Results indicate that the MCS-SVM is faster than the MCS, providing a suitable solution accuracy. Furthermore, the computational cost of the MCS-SVM is much less than the MCS when considering a large-scale structure. Nevertheless, the results demonstrate that increasing the number of features is more effective than increasing the size of the structure in reducing the computational efficiency of MCS-SVM. The reason is that the number of features is an essential factor influencing the complexity of an SVM regression model. Obviously, the MCS-SVM method is not limited to the probabilistic analysis of trusses and can also be applied to other skeletal structures such as frames.

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